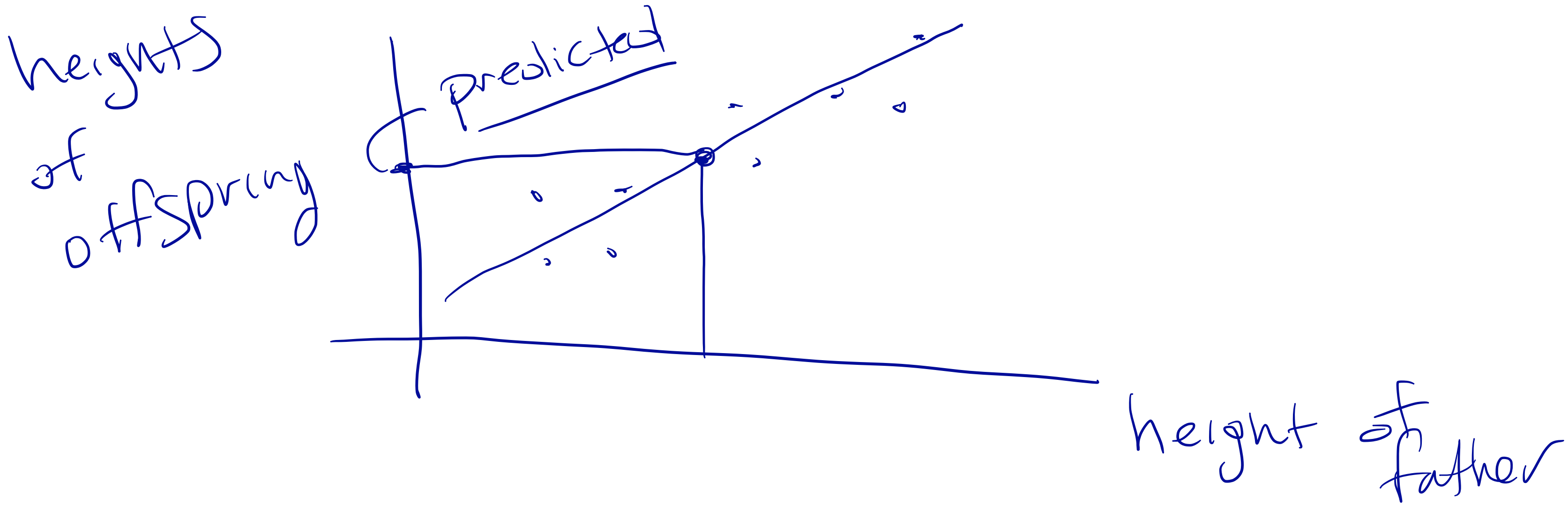
WB116 - STA130 2pm Section

Visiting data Scientist from

Geotab. - final project data.



Class 9 - Linear Regression

- In Classification Trees. the outcome Variable is binary (e.g., Yes, No or positive, Negative).

- Today we will discuss a prediction model for a continuous outcome. (e.g., height, weight, rating)

This Class

- Relationships between two variables
- Linear Relationships: The equation of a straight line
- Relationships between two variables
- Linear regression models
- Estimating the coefficients: Least Squares
- Interpreting the slope with a continuous explanatory variable
- Prediction/Supervised learning using a linear regression model
- R^2 - Coefficient of Determination
- Introduction to Multiple Regression
- RMSE – Root Mean Square Error as a measure of prediction accuracy.

Relationships between two variables

Advertising Example

- Suppose that we are statistical consultants hired by a client to provide advice on how to improve sales of a particular product.
- The `Advertising` data set consists of the sales of that product in 200 different markets, along with advertising budgets for the product in each of those markets for three different media: TV, radio, and newspaper.

```
glimpse(Advertising)
```

```
## Observations: 200
## Variables: 4
## $ TV        <dbl> 230.1, 44.5, 17.2, 151.5, 180.8, 8.7, 57.5, 120.2, 8...
## $ radio     <dbl> 37.8, 39.3, 45.9, 41.3, 10.8, 48.9, 32.8, 19.6, 2.1,...
## $ newspaper <dbl> 69.2, 45.1, 69.3, 58.5, 58.4, 75.0, 23.5, 11.6, 1.0,...
## $ sales     <dbl> 22.1, 10.4, 9.3, 18.5, 12.9, 7.2, 11.8, 13.2, 4.8, 1...
```

(features)
Independent variable

dependent variable
(target variables)

Budget spent of advertising

Sales.

Advertising Example

- It is not possible for our client to directly increase sales of the product, but they can control the advertising expenditure in each of the three media.
- Therefore, if we determine that there is an association between advertising and sales, then we can instruct our client to adjust advertising budgets, thereby indirectly increasing sales.

Increasing sales through advertising

What is the relationship between `sales` and `TV` budget?

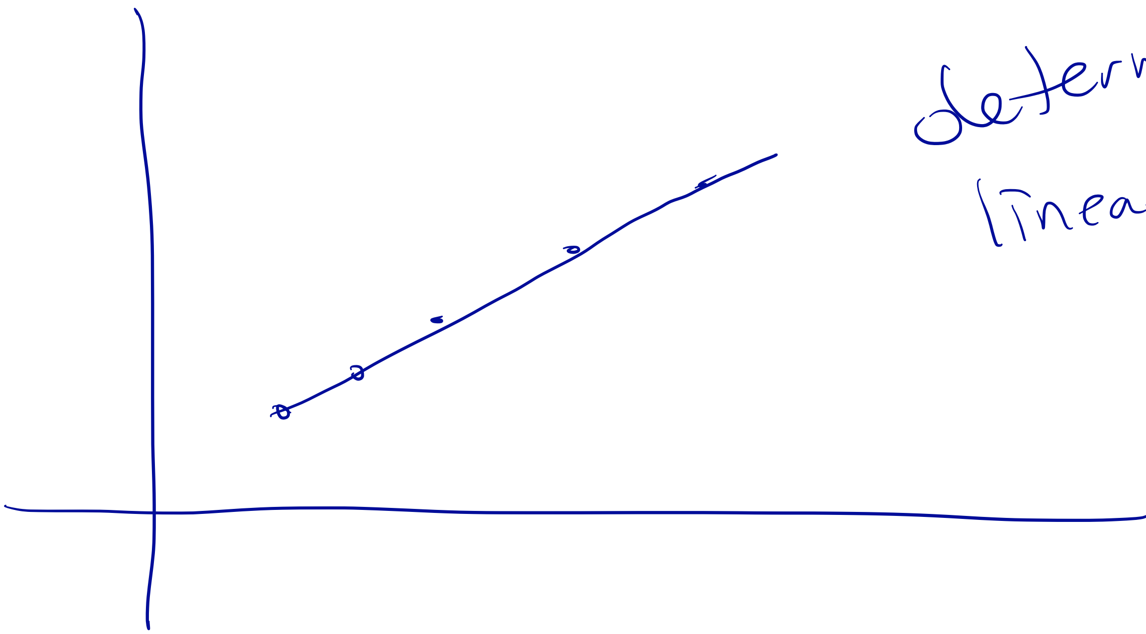
```
Advertising %>% ggplot(aes(x = TV, y = sales)) + geom_point() + theme_minimal()
```

As TV Budget increases Sales increases

$\therefore$ positive linear relationship.

Increasing sales through advertising

- In general, as the budget for `TV` increases `sales` increases.
- Although, sometimes increasing the `TV` budget didn't increase `sales`.
- The relationship between these two variables is approximately linear.



deterministic

linear relationship.

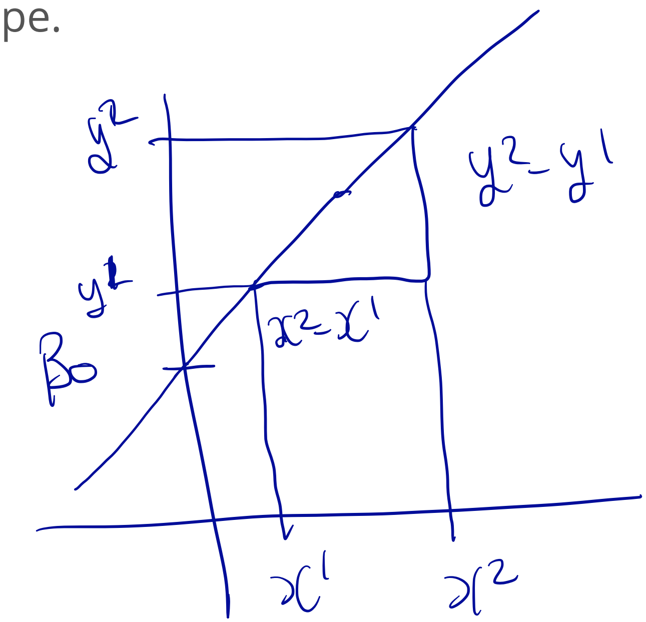
Linear Relationships

A perfect linear relationship between an independent variable x and dependent variable y has the mathematical form:

$$y = \beta_0 + \beta_1 x.$$

~~iep~~ β_0 is called the y-intercept and β_1 is called the slope.

at $x = 0$ $\quad y = \beta_0$



$$\beta_1 = \frac{y^2 - y^1}{x^2 - x^1}$$

Linear Relationships: The equation of a straight line

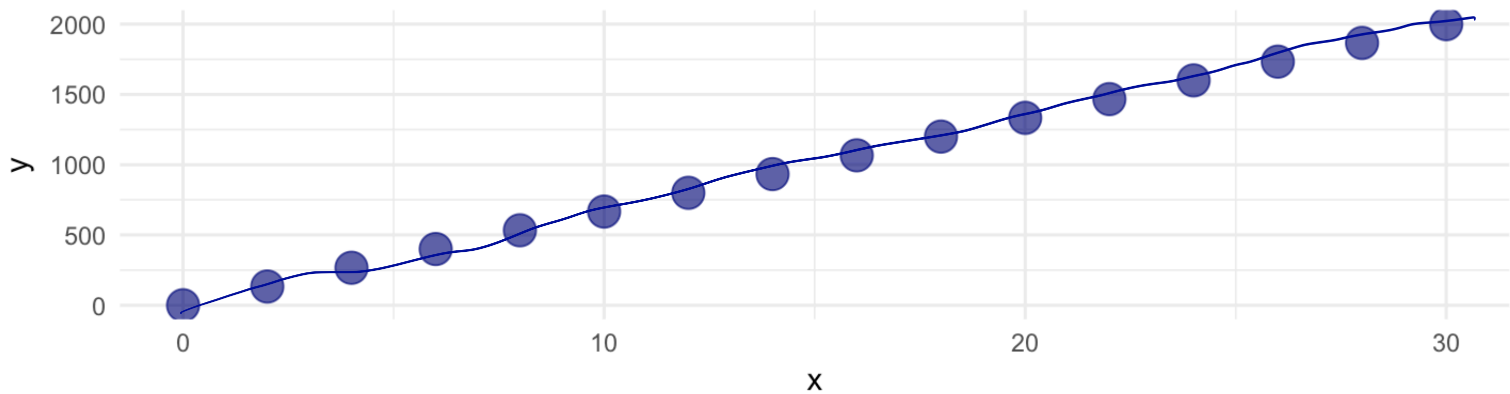
Linear Relationships: The equation of a straight line

If the relationship between y and x is perfectly linear then the scatter plot could look like:

Linear Relationships: The equation of a straight line

What is the equation of straight line that fits these points?

$y = \frac{133}{2} \cdot x$



First four observations:

```
## # A tibble: 4 x 2
##       x     y
##   <dbl> <dbl>
## 1  0        0
## 2  2.00   133
## 3  4.00   267
## 4  6.00   400
```

$m = \frac{133-0}{2-0} = 133/2$

$b = 0$

$y = \frac{133}{2} x$

Fitting a straight line to data

Use analytic geometry to find the equation of the straight line: pick two any points $(x^{(1)}, y^{(1)})$ and $(x^{(2)}, y^{(2)})$ on the line.

The slope is:

$$m = \frac{y^{(1)} - y^{(2)}}{x^{(1)} - x^{(2)}}.$$

So the equation of the line with slope m passing through $(x^{(1)}, y^{(1)})$ is

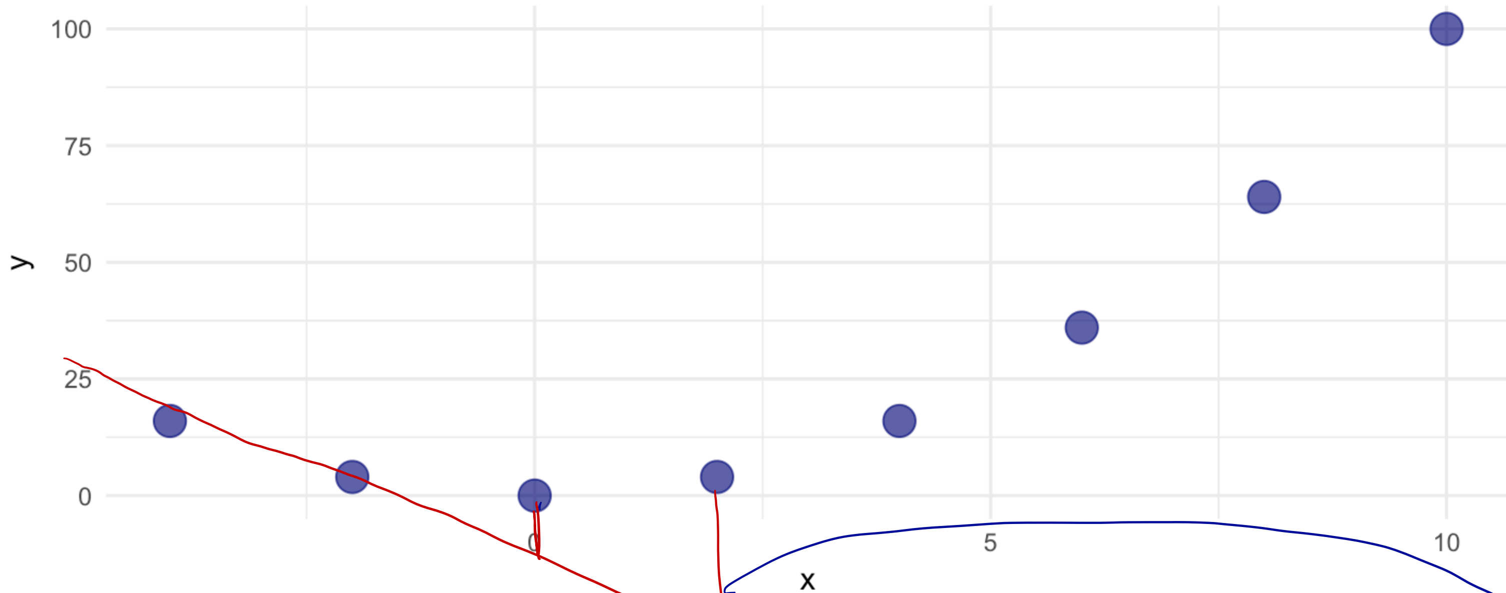
$$y - y^{(1)} = m(x - x^{(1)}) \Rightarrow y = mx + b,$$

where $b = y^{(1)} - mx^{(1)}$.

Linear Relationships: The equation of a straight line

- Relationship is Clearly non-linear
- Can still fit a straight line.
- But, it doesn't Capture the relationship very well.

What is the equation of the 'best' straight line that fits these points?




```
## # A tibble: 4 x 2
##       x     y
##   <dbl> <dbl>
## 1 -4.00 16.0
## 2 -2.00  4.00
## 3  0      0
```

$m = \frac{16-4}{-4-(-2)} = -6$

$-6x - 8 = y$

$y - (-16) = -6(x-(-4))$

Relationships between two variables

Relationships between two variables

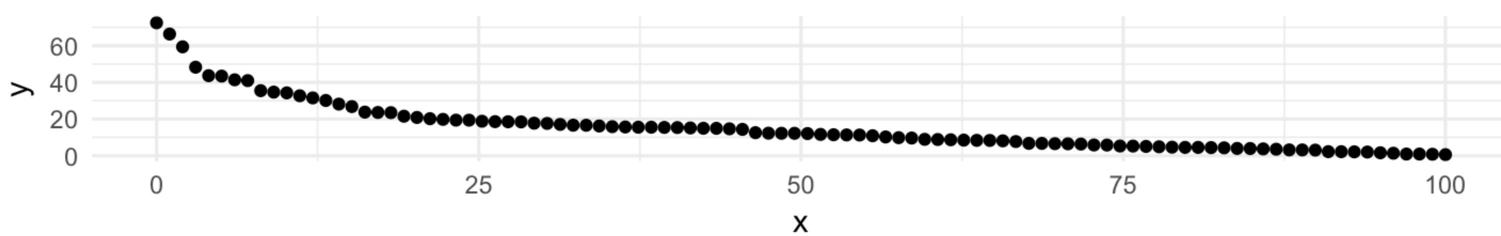
- Sometimes the relationship between two variables in non-linear.
- If the realtionship is non-linear then fitting a straight line to the data is not useful in describing the relationship.

Example of Non-linear relationships

- Let y be life expectancy of a component, and x the age of the component.
- There is a relationship between y and x, but it is not linear.

data is put into data frame

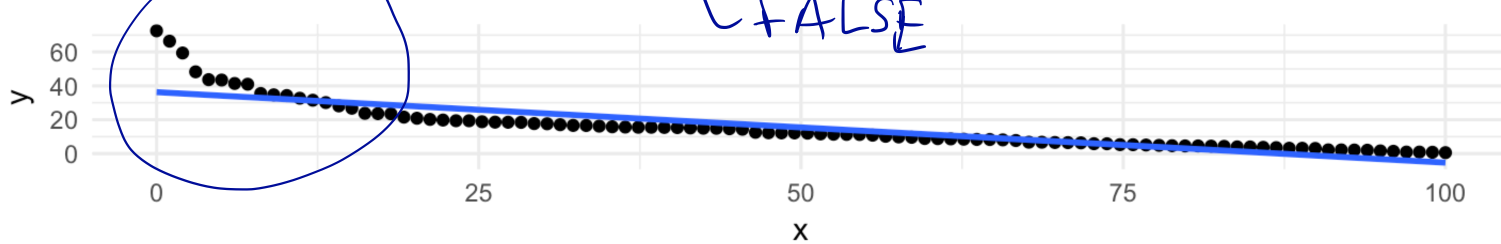
```
p <- data_frame(x = age, y = life_exp) %>%
  ggplot(aes(x = x, y = y)) + geom_point() + theme_minimal()
p
```



add a linear regression line to the scatter plot.

```
p + geom_smooth(method = "lm", se = F)
```

FALSE



Tidy the Advertising Data

- Each market is an observation, but each column is the amount spent on TV, radio, newspaper advertising.

```
## # A tibble: 3 x 4
##       TV radio newspaper sales
##    <dbl> <dbl>     <dbl> <dbl>
## 1 230     37.8      69.2  22.1
## 2  44.5   39.3      45.1  10.4
## 3  17.2   45.9      69.3   9.30
```

x_{11} x_{12} x_{13}

x_{23} x_{22} x_{21}

- The data are not tidy since each column corresponds to the values of advertising budget for different media.

the amount spent on TV, Radio, newspaper should be in a column called "amount" and another variable could be created to capture the advertising medium (TV, radio, newspaper).

Tidy the Advertising Data

- Tidy the data by creating a column for advertising budeget and another column for type of advertising.
- We can use the `gather` function in the `tidyr` library (part of the `tidyverse` library) to tidy the data.

```
Advertising_long <- Advertising %>%
  select(TV, radio, newspaper, sales) %>%
  gather(key = adtype, value = amount, TV, radio, newspaper)
head(Advertising_long)
```

```
## # A tibble: 6 x 3
##   sales adtype amount
##   <dbl> <chr>   <dbl>
## 1 22.1  TV      230
## 2 10.4  TV       44.5
## 3  9.30 TV       17.2
## 4 18.5  TV      152
## 5 12.9  TV      181
## 6  7.20 TV        8.70
```

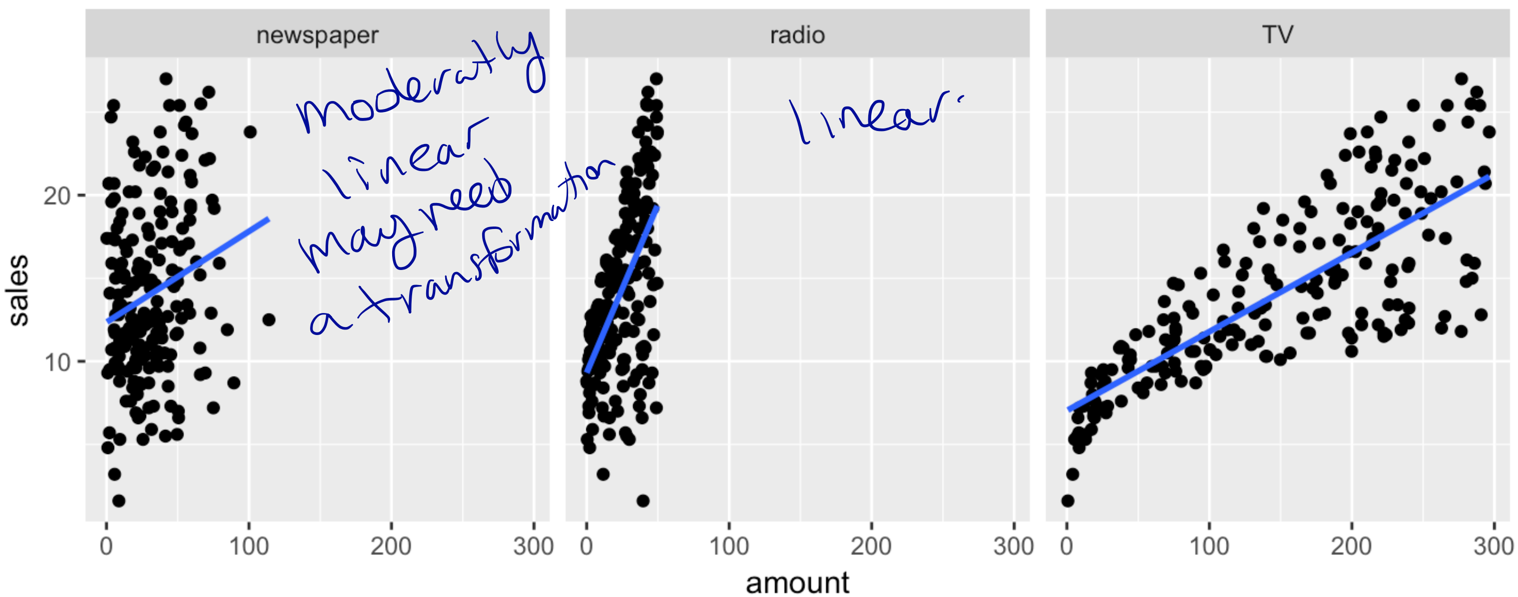
Sales TV radio newspaper

is often Called wide-data format

long-data format.

Advertising Data

```
Advertising_long %>%
  ggplot(aes(amount, sales)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  facet_grid(. ~ adtype)
```



linear.

- The advertising budgets (`newspaper`, `radio`, `TV`) are the input/independent/covariates and the dependent variable is sales.

Linear Regression Models

Simple Linear Regression

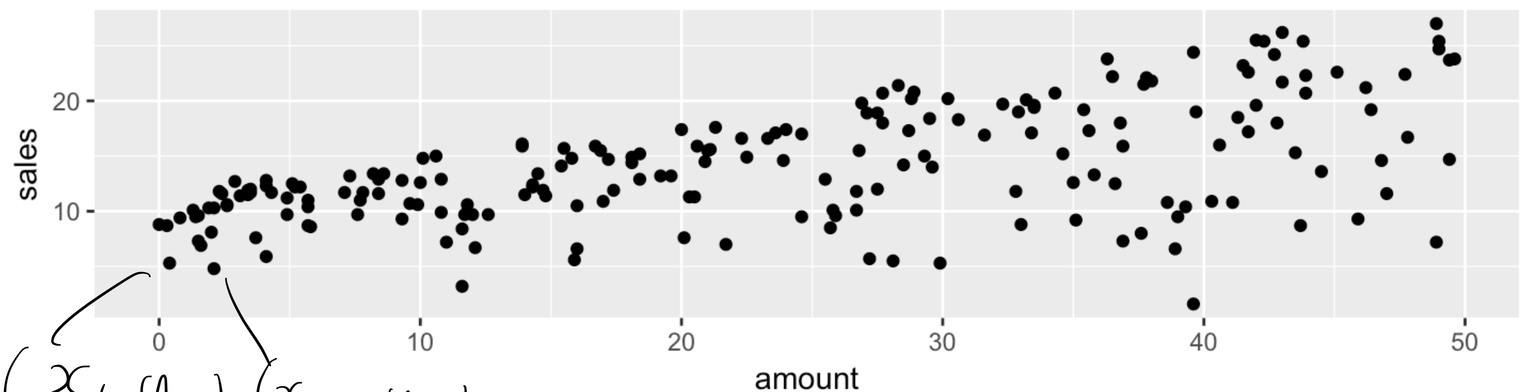
The simple linear regression model can describe the relationship between sales and amont spent on radio advertising through the model

Using the long data set.
$n = 200$

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

where $i = 1, \ldots, n$ and n is the number of observations.

```
Advertising_long %>%
  filter(adtype == "radio") %>%
  ggplot(aes(amount, sales)) +
  geom_point()
```



(x_1, y_1) (x_2, y_2)

x

Simple Linear Regression

The equation:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

is called a **regression model** and since we have only one independent variable it is called a *simple regression model*.

- y_i is called the dependent or target variable.
- β_0 is the intercept parameter.
- x_i is the independent variable, covariate, feature, or input.
- β_1 is called the slope parameter.
- ϵ_i is called the error parameter.

Statistical parameters.

Multiple Linear Regression

In general, models of the form

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \epsilon_i,$$

where $i = 1, \ldots, n$, with $k > 1$ independent variables are called *multiple regression models*.

- The β_j 's are called parameters and the ϵ_i 's errors.
- The values of of neither β_j 's nor ϵ_i 's can ever be known, but they can be estimated.
- The "linear" in Linear Regression means that the equation is linear in the parameters β_j .
- This is a linear regression model: $y_i = \beta_0 + \beta_1 \sqrt{x_{i1}} + \beta_2 x_{i2}^2 + \epsilon_i$
- This is not a linear regression model (i.e., a nonlinear regression model): $y_i = \beta_0 + \sin(\beta_1) x_{i1} + \beta_2 x_{i2} + \epsilon_i$

non-linear function of β_1

$$y_i = \beta_0 + \beta_1 \sqrt{x_{i1}} + \beta_2 x_{i2}^2 + \varepsilon_i$$

$i = 1, \ldots, 200$ where 200 is the number of observations in sales data (i.e., number of markets).

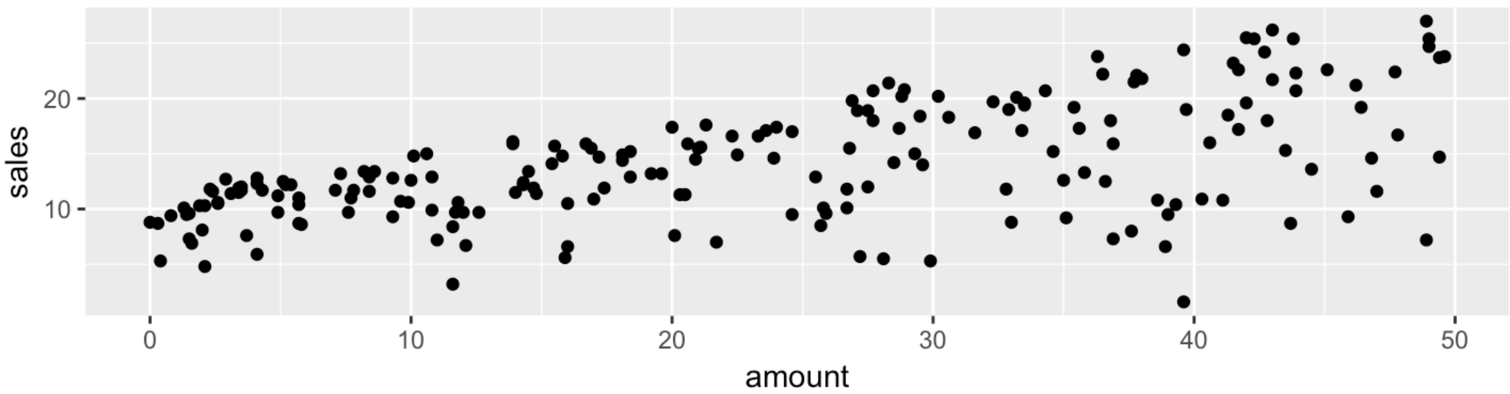
market	Sales	x_1 radio	x_2 newspaper	x_3 TV
1	y_1	x_{11}	x_{12}	x_{13}
2	y_2	x_{21}	x_{22}	x_{23}
3	y_3	x_{31}	x_{32}	x_{33}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
200				

$$y = 5x + 3$$

linear in x.

Least Squares

Fitting a straight line to Sales and Radio Advertising




```
## # A tibble: 6 x 2
##   sales amount
##   <dbl>  <dbl>
## 1 22.1    37.8
## 2 10.4    39.3
## 3  9.30   45.9
## 4 18.5    41.3
## 5 12.9    10.8
## 6  7.20   48.9
```

Fitting a straight line to Sales and Radio Advertising

```
head(Advertising_long %>%
  filter(adtype == "radio")) %>%
  select(sales,amount)
```

```
## # A tibble: 6 x 2
##   sales amount
##   <dbl>  <dbl>
## 1 22.1    37.8
## 2 10.4    39.3
## 3  9.30   45.9
## 4 18.5    41.3
## 5 12.9    10.8
## 6  7.20   48.9
```

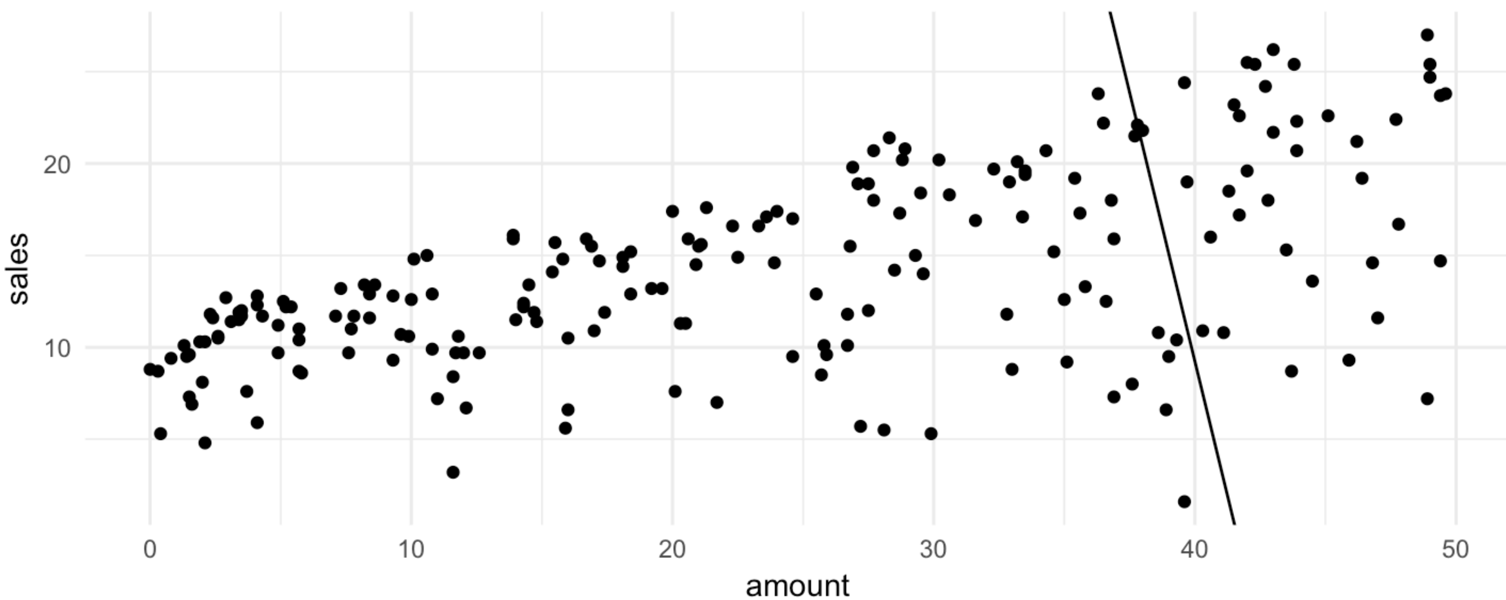
$m = \frac{22.1-10.4}{37.8-39.8} =$ -5.85, $b = 22.1 - \frac{22.1-10.4}{37.8-39.8} \times 37.8 =$ 243.23. So, the equation of the straight line is:

$$y = 243.23 - 5.85x.$$

Fitting a straight line to Sales and Radio Advertising

The equation $y = 243.23 - 5.85x$ is shown on the scatter plot.

this Can't be a good approach to approx. the relationship



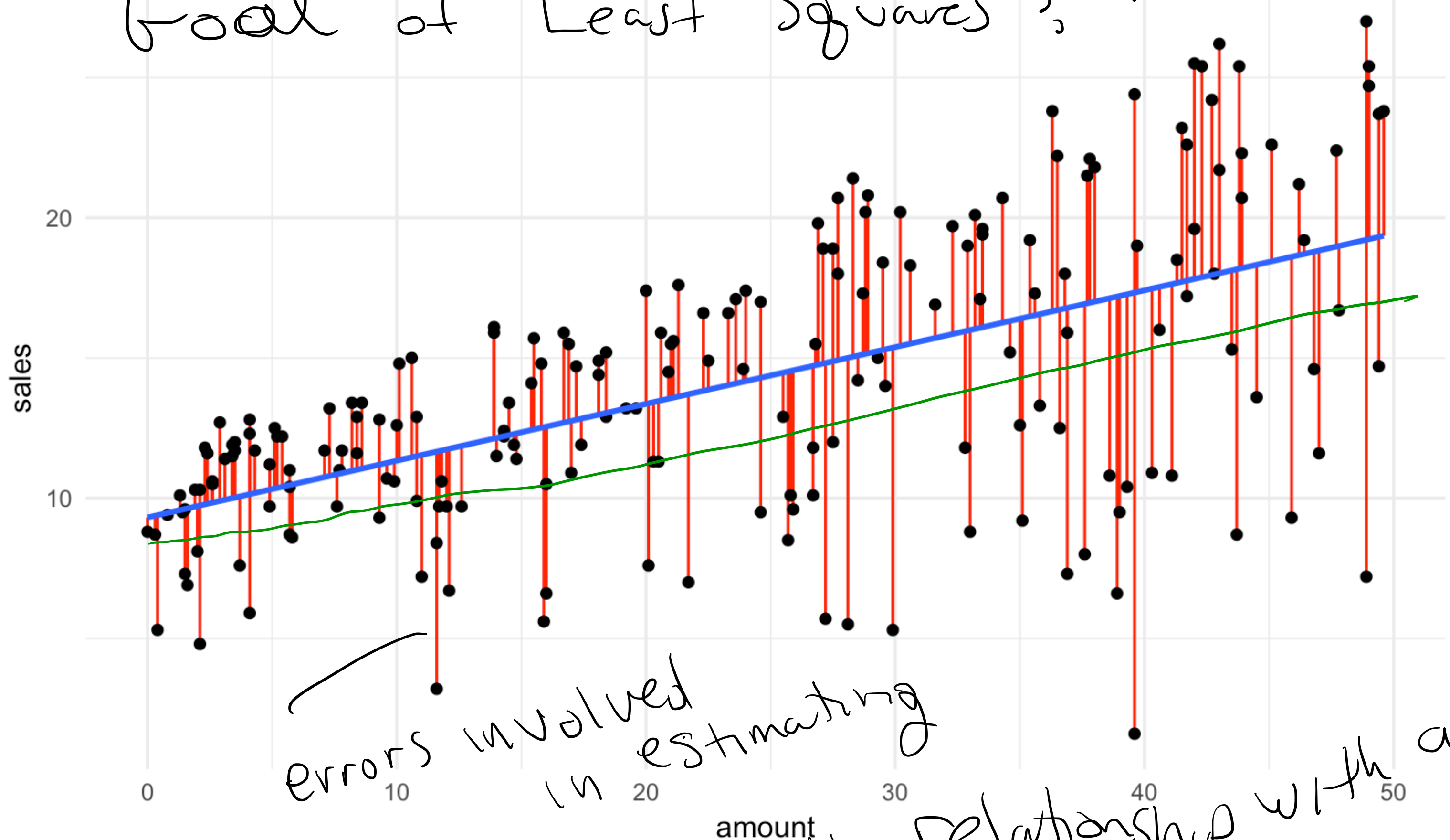
Fitting a straight line to Sales and Radio Advertising

- For a fixed value of `amount` spent on radio ads the corresponding `sales` has variation. It's neither strictly increasing nor decreasing.
- But, the overall pattern displayed in the scatterplot shows that *on average* `sales` increase as `amount` spent on radio ads increases.

Least Squares

The Least Squares approach is to find the y-intercept β_0 and slope β_1 of the straight line that is closest to as many of the points as possible.

Goal of Least Squares: minimize the errors.



errors involved in estimating the relationship with a straight line.

Estimating the coefficients: Least Squares

To find the values of β_0 and slope β_1 that fit the data best we can minimize the sum of squared errors $\sum_{i=1}^{n} \epsilon_i^2$:

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

errors

So, we want to minimize a function of β_0, β_1

$$L(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2,$$

$$\varepsilon_i = y_i - (\beta_0 + \beta_1 x_i)$$

$$\varepsilon_i^2 = \left(y_i - (\beta_0 + \beta_1 x_i)\right)^2$$

where x_i's are numbers and therfore constants.

minimize the sum of squared errors

$$\sum_{i=1}^{n} \varepsilon_i^2$$

Estimating the coefficients: Least Squares

- The derivative of $L(\beta_0, \beta_1)$ with respect to β_0 treats β_1 as a constant. This is also called the partial derivative and is denoted as $\frac{\partial L}{\partial \beta_0}$.

- To find the values of β_0 and β_1 that minimize $L(\beta_0, \beta_1)$ we set the partial derivatives to zero and solve:

$\frac{\partial L}{\partial \beta_0}$

$\frac{\partial L}{\partial \beta_1}$

$$\frac{\partial L}{\partial \beta_0} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0,$$

$$\frac{\partial L}{\partial \beta_0} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) x_i = 0.$$

The values of β_0 and β_1 that are solutions to above equations are denoted $\hat{\beta}_0$ and $\hat{\beta}_1$ respectively.

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$L(\beta_0, \beta_1) = \sum_{i=1}^{n} \left(y_i - \beta_0 - \beta_1 x_i\right)^2$$

minimize Squared error to find estimates of β_0, β_1

Now, take derivative w.r.t. β_0

$$\frac{d\,L(\beta_0, \beta_1)}{d\beta_1} = \frac{\partial L}{\partial \beta_0} = \frac{d}{d\beta_0}\left(\sum_{i=1}^{n}\left(y_i - \beta_0 - \beta_1 x_i\right)^2\right)$$

$$= \sum_{i=1}^{n} \frac{d}{d\beta_0}\left(y_i - \beta_0 - \beta_1 x_i\right)^2$$

$$= \sum_{i=1}^{n} 2\left(y_i - \beta_0 - \beta_1 x_i\right)(-1)$$

Set $\dfrac{\partial L}{\partial \beta_0} = 0$

Solve for $\hat{\beta}_0$

$$\sum_{i=1}^{n} 2(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)(-1) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\Rightarrow \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_0 - \sum_{i=1}^{n} \hat{\beta}_1 x_i = 0$$

$$\Rightarrow \sum_1^n y_i \underbrace{- n\hat{\beta}_0}_{\text{sum a constant } n \text{ times.}} - \hat{\beta}_1 \sum_1^n x_i = 0$$

$$\Rightarrow \sum_1^n y_i - \hat{\beta}_1 \sum x_i = n\hat{\beta}_0$$

$$\Rightarrow \frac{\sum_1^n y_i - \hat{\beta}_1 \sum x_i}{n} = \hat{\beta}_0 \Rightarrow \bar{y} - \hat{\beta}_1 \bar{x} = \hat{\beta}_0$$

$\bar{y} = \sum_1^n y_i / n$, $\bar{x} = \sum x_i / n$

Use $\hat{\beta}_0$, $\hat{\beta}_1$ to indicate that these are the values that are solutions to these two equations

Estimating the coefficients: Least Squares

It can be shown that

estimate of y-intercept

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{(\sum_{i=1}^{n} y_i x_i) - n\bar{x}\bar{y}}{(\sum_{i=1}^{n} x_i^2) - n\bar{x}^2},$$

— estimate of slope.

where, $\bar{y} = \sum_{i=1}^{n} y_i/n$, and $\bar{x} = \sum_{i=1}^{n} x_i/n$.

$\hat{\beta}_0$ and $\hat{\beta}_1$ are called the least squares estimators of β_0 and β_1.

Estimating the Coefficients Using R - Formula syntax in R

The R syntax for defining relationships between inputs such as amount spent on `newspaper` advertising and outputs such as `sales` is:

```
sales ~ newspaper
```

also used in rpart.

The tilde `~` is used to define the what the output variable (or outcome, on the left-hand side) is and what the input variables (or predictors, on the right-hand side) are.

A formula that has three inputs can be written as

```
sales ~ newspaper + TV + radio
```

dependent

independents.

Estimating the Coefficients Using `lm()`

linear model

```
mod_paper <- lm(sales ~ newspaper, data = Advertising)
mod_paper_summary <- summary(mod_paper)
mod_paper_summary$coefficients
```

```
##               Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 12.3514071 0.62142019 19.876096 4.713507e-49
## newspaper    0.0546931 0.01657572  3.299591 1.148196e-03
```

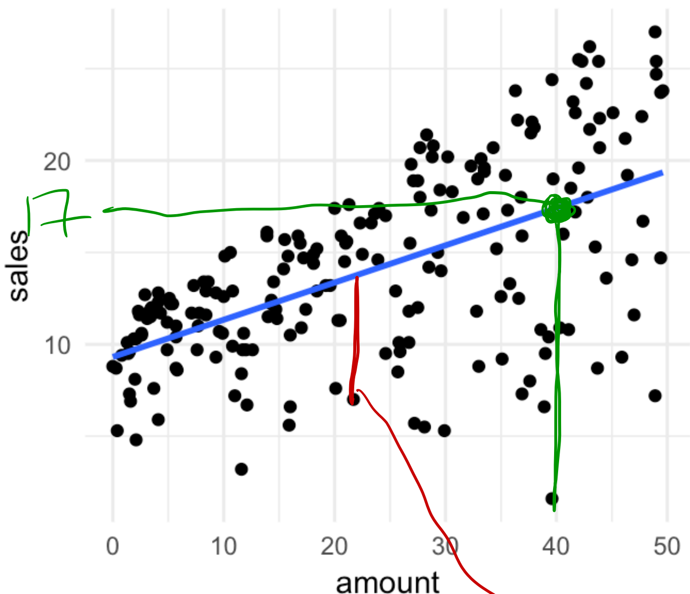
$\hat{\beta}_1$

- `(Intercept)` is the estimate of $\hat{\beta}_0$.
- `newspaper` is the estimate of $\hat{\beta}_1$.

$\hat{\beta}_0$

Estimating the Coefficients Using R

```
Advertising_long %>%
  filter(adtype == "radio") %>%
  ggplot(aes(amount, sales)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  theme_minimal()
```



the predicted Sales at radio at amount = 40 is ≃ 17

distance from observed to predicted is called residual.

- The blue line is the estimated regression line with intercept 12.35 and slope 0.05.
- `geom_smooth(method = "lm", se = FALSE)` adds the linear regression to the

Interpreting the Slope and Intercept with a Continuous Explanatory Variable

The estimated linear regression of `sales` on `newspaper` is:

$$y_i = 12.35 + 0.05x_i,$$

where y_i is sales in the i^{th} market and x_i is the dollar amount spent on newspaper advertising in the i^{th} market.

- The **slope** $\hat{\beta}_1$ is the amount of change in y for a unit change in x.
- Sales increase by 0.05 for each dollar spent on advertising.
- The **intercept** $\hat{\beta}_0$ is the average of y when $x_i = 0$.
- The average sales is 12.35 when the amount spent on advertising is zero.

Prediction using a Linear Regression Model

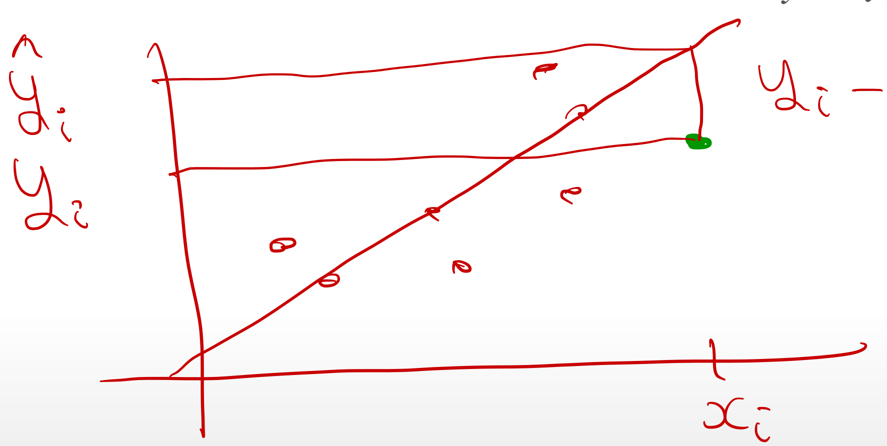
After a linear regression model is estimated from data it can be used to calculate predicted values using the regression equation

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i .$$

$\hat{y}_i$ is the predicted value of the i^{th} response y_i .

The i^{th} residual is

$$e_i = y_i - \hat{y}_i .$$



$y_i - \hat{y}_i = e_i$

Prediction using a Linear Regression Model

The amount spent on newspaper advertising in the first market is:

```
Advertising %>% filter(row_number() == 1)
```

```
## # A tibble: 1 x 4
##      TV radio newspaper sales
##   <dbl> <dbl>     <dbl> <dbl>
## 1   230  37.8      69.2  22.1
```

$\hat{y}_i = 12.35 + .05 x_i$

$\hat{y}_i = 12.35 + .05(69.2)$

- The predicted sales using the regression model is: $12.35 + 0.05 \times 69.2 =$ 16.14.
- The observed sales for region is 22.1.
- The **error** or **residual** is $y_1 - \hat{y}_1 =$ 5.96.

Prediction using a Linear Regression Model

The predicted and residual values from a regression model can be obtained using the `predict()` and `residual()` functions.

```
mod_paper <- lm(sales ~ newspaper, data = Advertising)
sales_pred <- predict(mod_paper)
head(sales_pred)
```

```
##        1        2        3        4        5        6
## 16.13617 14.81807 16.14164 15.55095 15.54548 16.45339
```

```
sales_resid <- residuals(mod_paper)
head(sales_resid)
```

```
##         1         2         3         4         5         6
##  5.963831 -4.418066 -6.841639  2.949047 -2.645484 -9.253389
```

Measure of Fit for Simple Regression

- The regression model is a good fit when the residuals are small.
- Thus, we can measure the quality of fit by the sum of squares of the residuals $\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$.
- This quantity depends on the units in which y_i's are measured. A measure of fit that does not depend on the units is:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}.$$

- R^2 is often called the coeffcient of determination.
- $0 \leq R^2 \leq 1$, where 1 indicates a perfect match between the observed and predicted values and 0 indicates an poor match.

$R^2 = 1 = 1 - 0 \iff e_i^2 = 0$

$(y_i - \hat{y}_i) = 0 \ \forall i.$

Measure of Fit for Simple Regression

The `summary()` method calculates R^2

```
mod_paper <- lm(sales ~ newspaper, data = Advertising)
mod_paper_summ <- summary(mod_paper)
mod_paper_summ$r.squared
```

```
## [1] 0.05212045
```

- $R^2 = 0.0521204$. This indicates a poor fit.

Using Linear Regression as a Machine Learning/Supervised Learning Tool

The `diamonds` data set contains the prices and other attributes of almost 54,000 diamonds. The variables are as follows:

```
## Observations: 53,940
## Variables: 10
## $ carat   <dbl> 0.23, 0.21, 0.23, 0.29, 0.31, 0.24, 0.24, 0.26, 0.22, ...
## $ cut     <ord> Ideal, Premium, Good, Premium, Good, Very Good, Very G...
## $ color   <ord> E, E, E, I, J, J, I, H, E, H, J, J, F, J, E, E, I, J, ...
## $ clarity <ord> SI2, SI1, VS1, VS2, SI2, VVS2, VVS1, SI1, VS2, VS1, SI...
## $ depth   <dbl> 61.5, 59.8, 56.9, 62.4, 63.3, 62.8, 62.3, 61.9, 65.1, ...
## $ table   <dbl> 55, 61, 65, 58, 58, 57, 57, 55, 61, 61, 55, 56, 61, 54...
## $ price   <int> 326, 326, 327, 334, 335, 336, 336, 337, 337, 338, 339,...
## $ x       <dbl> 3.95, 3.89, 4.05, 4.20, 4.34, 3.94, 3.95, 4.07, 3.87, ...
## $ y       <dbl> 3.98, 3.84, 4.07, 4.23, 4.35, 3.96, 3.98, 4.11, 3.78, ...
## $ z       <dbl> 2.43, 2.31, 2.31, 2.63, 2.75, 2.48, 2.47, 2.53, 2.49, ...
```

Question: Predict the price of diamonds based on carot size.

Predicting the Price of Diamonds

Let's select training and test sets.

```
set.seed(2)
diamonds_train <- diamonds %>%
  mutate(id = row_number()) %>%
  sample_frac(size = 0.2)

diamonds_test <- diamonds %>%
  mutate(id = row_number()) %>%
  # return all rows from diamonds where there are not
  # matching values in diamonds_train, keeping just
  # columns from diamonds.
  anti_join(diamonds_train, by = 'id')
```

training set 20% of data

test set

Predicting the Price of Diamonds

- Now fit a regression model on `diamonds_train`.

```
mod_train <- lm(price ~ carat, data = diamonds_train)
mod_train_summ <- summary(mod_train)
mod_train_summ$r.squared
```

```
## [1] 0.848017
```

Indicates good fit.

- Evaluate the prediction error using root mean square error using the training model on `diamonds_test`.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

- RMSE can be used to compare different sizes of data sets on an eual footing and the square root ensures that RMSE is on the same scale as y.

Predicting the Price of Diamonds using Simple Linear Regression

- Calculate RMSE using test and training data.

Model fit on training data

predictions using test data.

```
y_test <- diamonds_test$price
yhat_test <- predict(mod_train, newdata = diamonds_test)
n_test <- length(diamonds_test$price)

# test RMSE
rmse <- sqrt(sum((y_test - yhat_test)^2) / n_test)
rmse
```

```
## [1] 1548.794
```

∴ the two numbers are close no evidence of overfitting.

```
y_train <- diamonds_train$price
yhat_train <- predict(mod_train, newdata = diamonds_train)
n_train <- length(diamonds_train$price)

# train RMSE
sqrt(sum((y_train - yhat_train)^2) / n_train)
```

```
## [1] 1547.65
```

Predicting the Price of Diamonds using Multiple Linear Regression

We will add other variables to the regression model to investigate if we can decrease the prediction error.

```
mrmod_train <- lm(price ~ carat + cut + color + clarity, data = diamonds_train)
mrmod_train_summ <- summary(mrmod_train)
mrmod_train_summ$r.squared
```

```
## [1] 0.9175932
```

↑ from using only Carat.

```
y_test <- diamonds_test$price
yhat_test <- predict(mrmod_train, newdata = diamonds_test)
n_test <- length(diamonds_test$price)
mr_rmse <- sqrt(sum((y_test - yhat_test)^2) / n_test)
mr_rmse
```

```
## [1] 1161.982
```

↓ from using only Carat.

- The simple linear regression model had $R^2 = 0.848017$ and RMSE = 1548.794103.

Why minimize $\sum_{i=1}^{n} e_i^2$ instead of

$\sum_{i=1}^{n} e_i$?

$y=x^2$ has a guarunteed min.